USA Mathematical Talent Search
Round 3 Grading Rubric
Year 29 — Academic Year 2017–2018
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NOTE TO GRADERS: The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/3/29:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each incorrect entry.

Problem 2/3/29:

Note: The algebraic steps in this problem are fairly straightforward, so we put a lot of emphasis on making sure the explanation covers all the important considerations.

1 point: Student uses the given information to come up with a useful equation involving a, b, c, and q.

2 points: Student performs a series of algebraic steps to obtain $q = \frac{b^2-ac}{a-2b+c}$.

1 point: Student explains why we cannot have $a - 2b + c = 0$.

1 point: Student notes that since a, b, and c are integers, q is rational.

Problem 3/3/29:

1 point: Student correctly describes the region the orange triangle travels over during its rotation (e.g., the three sectors of the circle, plus the three quadrilaterals such as OA_1PC_2).

1 point: Student correctly explains how to find the area of the three sectors, and performs the relevant steps to compute the numerical value for their area. Award this point if the student's conceptual steps and formulas are correct, but the student makes a minor computational error.

2 points: Student correctly explains how to find the area of the quadrilateral regions, and performs the relevant steps to compute the numerical value for the area. Award both points if the student's conceptual steps and formulas are correct, but the student makes a minor computational error. Award **1 point** of partial credit if the student makes significant constructive progress towards finding the area of the quadrilateral regions.

1 point: Student obtains the correct answer of $\frac{\sqrt{3}+\pi}{18}$, with or without explanation.

Note: A figure was essential for this problem. With rare exceptions, we awarded a maximum of **4 points** if the solution was missing a figure.

Problem 4/3/29:

5 points: Student provides (with justification) a valid example of a polynomial $P(x)$ with rational coefficients that takes on an integer value for each uphill positive integer x, but that does not take on an integer value for each integer x. Point values for key steps in the official solution are as follows:

1 point: Student recognizes the relevance of modular arithmetic to this problem.

1 point: Student recognizes that no uphill integer is congruent to 10 modulo 11.

3 points: Student identifies a polynomial $P(x)$ that takes on an integer value if and only x is not congruent to 10 modulo 11 with appropriate justification. Award **1 point** for stating the polynomial, and up to **2 points** for the justification. Award partial credit as appropriate depending on the quality of the justification.

Note: Award at least **1 point** of partial credit if the student gives an answer of "no" with enough explanation (correct or incorrect) that makes it clear that the student has some rationale for their answer (as opposed to having made a blind guess).

Problem 5/3/29:

1 point: Student obtains the correct answer, with or without justification.

1 point: Student proposes a viable strategy for solving the problem (e.g., student suggests a relevant set of cases to analyze, student suggests that we analyze how many ways there are to deal the next two cards if an odd number of cards have been dealt so far).

3 points: Student provides a complete and correct analysis using the strategy they propose. Award partial credit as appropriate. For the method in the official solution, these three points should be awarded as follows:

1 point: Student recognizes that if j is odd, there is exactly one card in the pit, and if j is even, then there are either zero or two cards in the pit.

1 point: Student recognizes that if we are at the odd position s_{2k-1}, then there are three ways to deal the next two cards to get s_{2k+1}.

1 point: Student recognizes that there is one way to deal the final card, and also recognizes the need to multiply by $2^n \cdot n!$.