

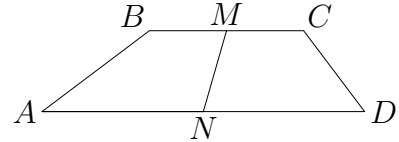


USA Mathematical Talent Search

Solutions to Problem 2/2/18

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2/2/18. $ABCD$ is a trapezoid with $\overline{BC} \parallel \overline{AD}$, $\angle ADC = 57^\circ$, $\angle DAB = 33^\circ$, $BC = 6$, and $AD = 10$. M and N are the midpoints of \overline{BC} and \overline{AD} , respectively.



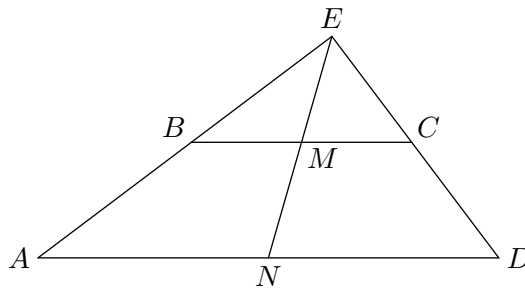
- (a) Find $\angle MNA$.
- (b) Find MN .

Credit This problem was proposed by Gregory Galperin.

Comments The fact that the angles 57° and 33° are complementary is a strong indication to “complete” the right triangle, as in the solution below. Once this right triangle is constructed, the required data is easy to calculate. The fact that lines AB , CD and MN are concurrent must be justified. *Solutions edited by Naoki Sato.*

Solution 1 by: David Corwin (10/MA)

Extend AB and DC to meet at point E . Because $BC \parallel AD$, segments BC and AD are homothetic with respect to point E , with ratio $\frac{AD}{BC} = \frac{10}{6} = \frac{5}{3}$. Because M is the midpoint of BC , its corresponding homothetic point on AD is the midpoint of AD , which is N , so M and N are homothetic with respect to point E , and therefore E , M , and N are collinear.



(a) By triangle AED , $\angle AED = 180^\circ - \angle EAD - \angle EDA = 180^\circ - 33^\circ - 57^\circ = 90^\circ$, so triangle AED is right. Because N is the midpoint of hypotenuse AD , $NE = AN$, so triangle ANE is isosceles, and $\angle NEA = \angle NAE = 33^\circ$. Then by triangle ANE , $\angle ANE = \angle ANM = 180^\circ - \angle NEA - \angle NAE = 180^\circ - 33^\circ - 33^\circ = 114^\circ$.

(b) Because N is the midpoint of AD , $NE = AN = \frac{AD}{2} = \frac{10}{2} = 5$. By the homothety, $ME = \frac{NE}{\frac{5}{3}} = \frac{5}{\frac{5}{3}} = 3$. Then $MN = NE - ME = 5 - 3 = 2$.