USA Mathematical Talent Search

Round 2 Problems

Year 30 — Academic Year 2018-2019

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by November 26, 2018, via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 8 PM Eastern / 5 PM Pacific on November 26, 2018.

   (b) Mail: USAMTS
   55 Exchange Place
   Suite 603
   New York, NY 10005
   Deadline: Solutions must be postmarked on or before November 26.

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto www.usamts.org and visiting the "My USAMTS" pages.

7. Round 2 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on www.usamts.org.

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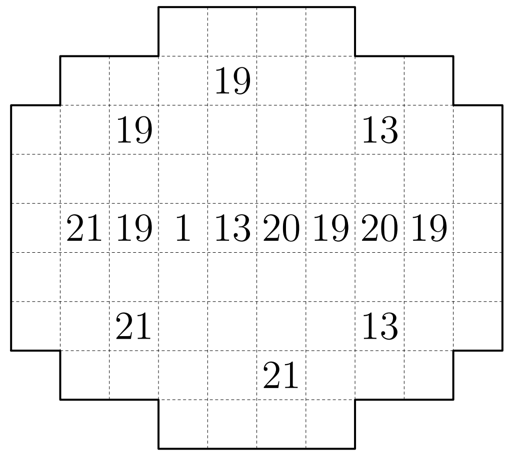
Each problem is worth 5 points.

1/2/30. The grid to the right consists of 74 unit squares, marked by gridlines. Partition the grid into five regions along the gridlines so that the areas of the regions are 1, 13, 19, 20, and 21. A square with a number should be contained in the region with that area.

You do not need to prove that your answer is the only one possible; you merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)



2/2/30. Given a set of positive integers R, we define the *friend set* of R to be all positive integers that are divisible by at least one number in R. The friend set of R is denoted by $\mathcal{F}(R)$. A set G is called *unfriendly* if no element of G is a divisor of another element of G.

Let S_1 and S_2 be unfriendly sets. Suppose that $\mathcal{F}(S_1) = \mathcal{F}(S_2)$. Show that $S_1 = S_2$.

3/2/30. Alice, Bob, and Chebyshev play a game. Alice puts six red chips into a bag, Bob puts seven blue chips into the bag, and Chebyshev puts eight green chips into the bag. Then, the almighty Zan randomly removes chips from the bag one at a time and gives them back to the corresponding player. The winner of the game is the first player to get all of their chips back. Find, with proof, the probability that Bob wins the game.

4/2/30. Find, with proof, all ordered pairs of positive integers (a, b) with the following property: there exist positive integers r, s, and t such that for all n for which both sides are defined,

$$\binom{\binom{n}{a}}{b} = r\binom{n+s}{t}.$$

5/2/30. Acute scalene triangle $\triangle ABC$ has circumcenter O and orthocenter H. Points X and Y, distinct from B and C, lie on the circumcircle of $\triangle ABC$ such that $\angle BXH = \angle CYH = 90°$. Show that if lines XY, AH, and BC are concurrent, then OH is parallel to BC.

Problems by Shyan Akmal, David Altizio, Michael Tang, Sam Vandervelde, and USAMTS Staff.

Round 2 Solutions must be submitted by **November 26, 2018**.

Please visit **http://www.usamts.org** for details about solution submission.