



USA Mathematical Talent Search

Round 1 Problems

Year 19 — Academic Year 2007–2008

www.usamts.org

Please follow the rules below:

1. You must show your work and prove your answers on all problems. If you just send a numerical answer for a problem with no proof, you will get no more than 1 point.
2. If you have not already sent an Entry Form, download an Entry Form from the Forms page at
http://www.usamts.org/MyUSAMTS/U_MyForms.php
and submit the completed form with your solutions.
3. If you have already sent in an Entry Form and a Permission Form, you do not need to resend them.
4. Put your name and USAMTS ID# on **every page you submit**.
5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.
6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging into www.usamts.org, then clicking on My USAMTS on the sidebar, then clicking Profile. If you are registered for the USAMTS and haven't received any email from us about the USAMTS, your email address is probably wrong in your Profile.
7. Do not fax solutions written in pencil.
8. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.
9. Round 1 results will be posted at www.usamts.org by mid-November. To see your results, log in to the USAMTS page, then go to My USAMTS. Check that your email address in your USAMTS Profile is correct; you will receive an email when the scores are available.
10. Submit your solutions by October 9, 2007 (postmark deadline), via one (and only one!) of the methods below.
 - (a) Email: solutions@usamts.org. Please see usamts.org for a list of acceptable file types. Do not send Microsoft Word files.
 - (b) Fax: (619) 445-2379 (You must include a cover sheet indicating the number of pages you are faxing, your name, and your USAMTS ID#.)
 - (c) Snail mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.
11. Re-read Items 1–10.



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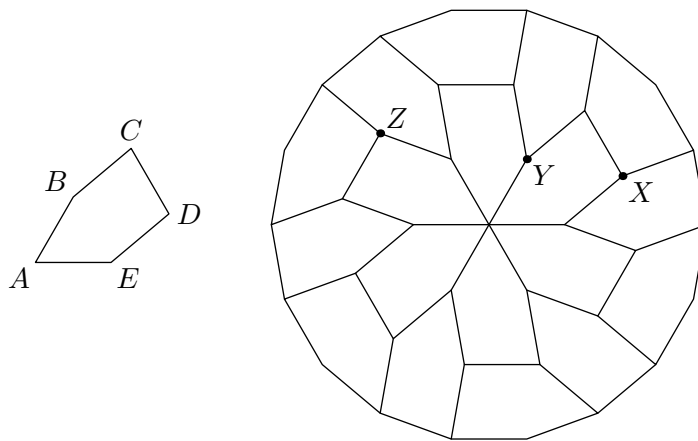
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1/1/19. Gene has $2n$ pieces of paper numbered 1 through $2n$. He removes n pieces of paper that are numbered consecutively. The sum of the numbers on the remaining pieces of paper is 1615. Find all possible values of n .

2/1/19. A regular 18-gon is dissected into 18 pentagons, each of which is congruent to pentagon $ABCDE$, as shown. All sides of the pentagon have the same length.



- (a) Determine angles A , B , C , D , and E .
- (b) Show that points X , Y , and Z are collinear.

3/1/19. Find all positive integers $a \leq b \leq c$ such that

$$\arctan \frac{1}{a} + \arctan \frac{1}{b} + \arctan \frac{1}{c} = \frac{\pi}{4}.$$

4/1/19. In convex quadrilateral $ABCD$, $AB = CD$, $\angle ABC = 77^\circ$, and $\angle BCD = 150^\circ$. Let P be the intersection of the perpendicular bisectors of \overline{BC} and \overline{AD} . Find $\angle BPC$.

5/1/19. Let c be a real number. The sequence a_1, a_2, a_3, \dots is defined by $a_1 = c$ and $a_n = 2a_{n-1}^2 - 1$ for all $n \geq 2$. Find all values of c such that $a_n < 0$ for all $n \geq 1$.

Round 1 Solutions must be submitted by **October 9, 2007**.

Please visit <http://www.usamts.org> for details about solution submission.

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