USA Mathematical Talent Search

Round 2 Problems

Year 19 — Academic Year 2007–2008

www.usamts.org

Please follow the rules below:

1. You must show your work and prove your answers on all problems. If you just send a numerical answer for a problem with no proof, you will get no more than 1 point.
2. If you have not already sent an Entry Form, download an Entry Form from the Forms page at

   `http://www.usamts.org/MyUSAMTS/U_MyForms.php`

   and submit the completed form with your solutions.
3. If you have already sent in an Entry Form and a Permission Form, you do not need to resend them.
4. Put your name and USAMTS ID# on **every page you submit**.
5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.
6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging into www.usamts.org, then clicking on My USAMTS on the sidebar, then clicking Profile. If you are registered for the USAMTS and haven't received any email from us about the USAMTS, your email address is probably wrong in your Profile.
7. Do not fax solutions written in pencil.
8. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.
9. Round 2 results will be posted at www.usamts.org by mid-January. To see your results, log in to the USAMTS page, then go to My USAMTS. Check that your email address in your USAMTS Profile is correct; you will receive an email when the scores are available.
10. Submit your solutions by November 19, 2007 (postmark deadline), via one (and only one!) of the methods below.
    - (a) Email: solutions@usamts.org. Please see usamts.org for a list of acceptable file types. Do not send Microsoft Word files.
    - (b) Fax: (619) 445-2379 (You must include a cover sheet indicating the number of pages you are faxing, your name, and your USAMTS ID#.)
    - (c) Snail mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.
11. Re–read Items 1–10.

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1/2/19. Find the smallest positive integer n such that every possible coloring of the integers from 1 to n with each integer either red or blue has at least one arithmetic progression of three different integers of the same color.

2/2/19. Let x, y, and z be complex numbers such that $x + y + z = x^5 + y^5 + z^5 = 0$ and $x^3 + y^3 + z^3 = 3$. Find all possible values of $x^{2007} + y^{2007} + z^{2007}$.

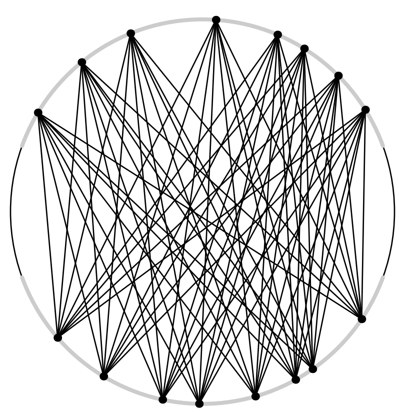
3/2/19. A triangular array of positive integers is called *remarkable* if all of its entries are distinct, and each entry, other than those in the top row, is the quotient of the two numbers immediately above it. For example, the following triangular array is remarkable:

$$\begin{array}{ccccc} 7 & & 42 & & 14 \\ & 6 & & 3 & \\ & & 2 & & \end{array}$$

Find the smallest positive integer that can occur as the greatest element in a remarkable array with four numbers in the top row.

4/2/19. Two nonoverlapping arcs of a circle are chosen. Eight distinct points are then chosen on each arc. All 64 segments connecting a chosen point on one arc to a chosen point on the other arc are drawn. How many triangles are formed that have at least one of the 16 points as a vertex?

A sample figure is shown below:



5/2/19. Faces ABC and XYZ of a regular icosahedron are parallel, with the vertices labeled such that $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$ are concurrent. Let $\mathcal{S}$ be the solid with faces ABC, AYZ, BXZ, CXY, XBC, YAC, ZAB, and XYZ. If $AB = 6$, what is the volume of $\mathcal{S}$?

Round 2 Solutions must be submitted by **November 19, 2007**.
Please visit **http://www.usamts.org** for details about solution submission.