

# U S A Mathematical Talent Search

## PROBLEMS

### Round 1 - Year 11 - Academic Year 1999-2000

- 1/1/11.** The digits of the three-digit integers  $a$ ,  $b$ , and  $c$  are the nine nonzero digits 1, 2, 3, ..., 9, each of them appearing exactly once. Given that the ratio  $a:b:c$  is 1:3:5, determine  $a$ ,  $b$ , and  $c$ .
- 2/1/11.** Let  $N = 111\dots1222\dots2$ , where there are 1999 digits of 1 followed by 1999 digits of 2. Express  $N$  as the product of four integers, each of them greater than 1.
- 3/1/11.** Triangle  $ABC$  has angle  $A$  measuring  $30^\circ$ , angle  $B$  measuring  $60^\circ$ , and angle  $C$  measuring  $90^\circ$ . Show four different ways to divide triangle  $ABC$  into four triangles, each similar to triangle  $ABC$ , but with one quarter of the area. Prove that the angles and sizes of the smaller triangles are correct.
- 4/1/11.** There are 8436 steel balls, each with radius 1 centimeter, stacked in a tetrahedral pile, with one ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. Determine the height of the pile in centimeters.
- 5/1/11.** In a convex pentagon  $ABCDE$  the sides have lengths 1, 2, 3, 4, and 5, though not necessarily in that order. Let  $F$ ,  $G$ ,  $H$ , and  $I$  be the midpoints of the sides  $AB$ ,  $BC$ ,  $CD$ , and  $DE$ , respectively. Let  $X$  be the midpoint of segment  $FH$ , and  $Y$  be the midpoint of segment  $GI$ . The length of segment  $XY$  is an integer. Find all possible values for the length of side  $AE$ .

\*\*\*\*\*

Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet and a completed Entry Form, should be sent to the following address and **post-marked no later than September 13, 1999**. Each participant is expected to develop solutions without help from others.

USA Mathematical Talent Search  
COMAP Inc., Suite 210  
57 Bedford Street  
Lexington, MA 02173