U S A Mathematical Talent Search

PROBLEMS
Round 1 - Year 12 - Academic Year 2000-2001

1/1/12. Determine the smallest five-digit positive integer N such that $2N$ is also a five-digit integer and all ten digits from 0 to 9 are found in N and $2N$.

2/1/12. It was recently shown that $2^{2^{24}} + 1$ is not a prime number. Find the four rightmost digits of this number.

3/1/12. Determine the integers a, b, c, d, and e for which

$$(x^2 + ax + b)(x^3 + cx^2 + dx + e) = x^5 - 9x - 27.$$

4/1/12. A sequence of real numbers $s_0, s_1, s_2, \ldots$ has the property that

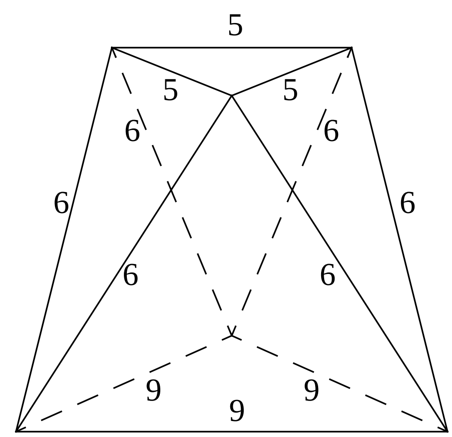
$s_i s_j = s_{i+j} + s_{i-j}$ for all nonnegative integers i and j with $i \geq j$,

$s_i = s_{i+12}$ for all nonnegative integers i, and

$s_0 > s_1 > s_2 > 0$.

Find the three numbers s_0, s_1, and s_2.

5/1/12. In the octahedron shown on the right, the base and top faces are equilateral triangles with sides measuring 9 and 5 units, and the lateral edges are all of length 6 units. Determine the height of the octahedron; i.e., the distance between the base and the top face.



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Complete, well-written solutions to at least two of the problems above, accompanied by a completed **Cover Sheet** and a completed **Entry Form** (both available on the web site http://www.nsa.gov/programs/mepp/usamts.html), should be sent to the following address and **postmarked no later than September 11, 2000**. Each participant is expected to develop solutions without help from others.

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