

U S A Mathematical Talent Search

PROBLEMS

Round 2 - Year 12 - Academic Year 2000-2001

1/2/12. A well-known test for divisibility by 19 is as follows: Remove the last digit of the number, add twice that digit to the truncated number, and keep repeating this procedure until a number less than 20 is obtained. Then, the original number is divisible by 19 if and only if the final number is 19. The method is exemplified on the right; it is easy to check that indeed 67944 is divisible by 19, while 44976 is not.

6 7 9 4 4	4 4 9 7 6
8	1 2
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
6 8 0 2	4 5 0 6
4	1 8
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
6 8 4	4 6 8
8	1 6
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
7 6	6 2
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
1 2	4
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
1 9	1 0

Find and prove a similar test for divisibility by 29.

2/2/12. Compute $1776^{1492!} \pmod{2000}$; i.e., the remainder when $1776^{1492!}$ is divided by 2000. (As usual, the exclamation point denotes factorial.)

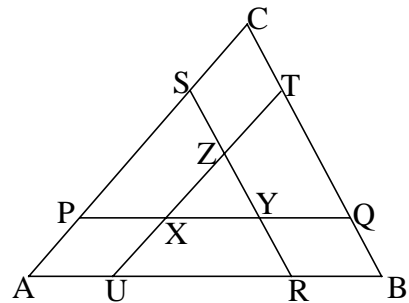
3/2/12. Given the arithmetic progression of integers
 $308, 973, 1638, 2303, 2968, 3633, 4298,$
determine the unique geometric progression of integers,
 $b_1, b_2, b_3, b_4, b_5, b_6$

so that
 $308 < b_1 < 973 < b_2 < 1638 < b_3 < 2303 < b_4 < 2968 < b_5 < 3633 < b_6 < 4298.$

4/2/12. Prove that every polyhedron has two vertices at which the same number of edges meet.
[Editor's comment: Proofs are difficult to grade. Please try to keep your solution short and well written.]

5/2/12. In $\triangle ABC$, segments PQ , RS , and TU are parallel to sides AB , BC , and CA , respectively, and intersect at the points X , Y , and Z , as shown in the figure on the right.

Determine the area of $\triangle ABC$ if each of the segments PQ , RS , and TU bisects (halves) the area of $\triangle ABC$, and if the area of $\triangle XYZ$ is one unit. Your answer should be in the form $a + b\sqrt{2}$, where a and b are positive integers.



Complete, well-written solutions to at least two of the problems above, accompanied by a completed **Cover Sheet** (available on the web site <http://www.nsa.gov/programs/mepp/usamts.html>), should be sent to the following address and **postmarked no later than November 13, 2000**. Also include an **Entry Form** if you did not participate in Round 1. Each participant is expected to develop solutions without help from others.

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