USA Mathematical Talent Search

PROBLEMS
Round 3 - Year 12 - Academic Year 2000-2001

1/3/12. Find the smallest positive integer with the property that it has divisors ending with every decimal digit; i.e., divisors ending in 0, 1, 2, ..., 9.

2/3/12. Assume that the irreducible fractions between 0 and 1, with denominators at most 99, are listed in ascending order. Determine which two fractions are adjacent to $\frac{17}{76}$ in this listing.

3/3/12. Let $p(x) = x^5 + x^2 + 1$ have roots r_1, r_2, r_3, r_4, r_5. Let $q(x) = x^2 - 2$. Determine the product $q(r_1)q(r_2)q(r_3)q(r_4)q(r_5)$.

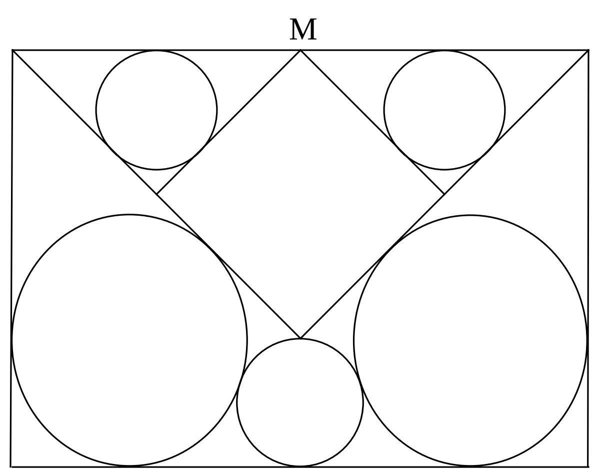
4/3/12. Assume that each member of the sequence $\langle \Diamond_i \rangle_{i=1}^{\infty}$ is either a + or a – sign. Determine the appropriate sequence of + and – signs so that

$$2 = \sqrt{6 \Diamond_1 \sqrt{6 \Diamond_2 \sqrt{6 \Diamond_3 \dots}}}.$$

Also determine what sequence of signs is necessary if the sixes in the nested roots are replaced by sevens. List all integers that work in the place of sixes and the sequences of signs that are needed with them.

5/3/12. Three isosceles right triangles are erected from the larger side of a rectangle into the interior of the rectangle, as shown on the right, where M is the midpoint of that side. Five circles are inscribed tangent to some of the sides and to one another as shown. One of the circles touches the vertex of the largest triangle.

Find the ratios among the radii of the five circles.



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Complete, well-written solutions to at least two of the problems above, accompanied by a completed **Cover Sheet** (available on the web site http://www.nsa.gov/programs/mepp/usamts.html), should be sent to the following address and **postmarked no later than January 8, 2001**. Also include an **Entry Form** if this is your first submission this academic year. Each participant is expected to develop solutions without help from others.

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