

USA Mathematical Talent Search

PROBLEMS

Round 3 - Year 14 - Academic Year 2002–2003

1/3/14. The integer n , between 10000 and 99999, is $abcde$ when written in decimal notation. The digit a is the remainder when n is divided by 2, the digit b is the remainder when n is divided by 3, the digit c is the remainder when n is divided by 4, the digit d is the remainder when n is divided by 5, and the digit e is the remainder when n is divided by 6. Find n .

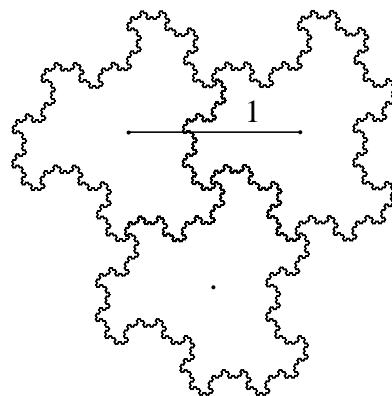
2/3/14. Given positive integers p , u , and v such that $u^2 + 2v^2 = p$, determine, in terms of u and v , integers m and n such that $3m^2 - 2mn + 3n^2 = 24p$. (It is known that if p is any prime number congruent to 1 or 3 modulo 8, then we can find integers u and v such that $u^2 + 2v^2 = p$.)

3/3/14. Determine, with proof, the rational number $\frac{m}{n}$ that equals

$$\frac{1}{1\sqrt{2} + 2\sqrt{1}} + \frac{1}{2\sqrt{3} + 3\sqrt{2}} + \frac{1}{3\sqrt{4} + 4\sqrt{3}} + \dots + \frac{1}{4012008\sqrt{4012009} + 4012009\sqrt{4012008}}.$$

4/3/14. The vertices of a cube have coordinates $(0, 0, 0)$, $(0, 0, 4)$, $(0, 4, 0)$, $(0, 4, 4)$, $(4, 0, 0)$, $(4, 0, 4)$, $(4, 4, 0)$, and $(4, 4, 4)$. A plane cuts the edges of this cube at the points $(0, 2, 0)$, $(1, 0, 0)$, $(1, 4, 4)$, and two other points. Find the coordinates of the other two points.

5/3/14. A fudgeflake is a planar fractal figure with 120° rotational symmetry such that three identical fudgeflakes in the same orientation fit together without gaps to form a larger fudgeflake with its orientation 30° clockwise of the smaller fudgeflakes' orientation, as shown on the right. If the distance between the centers of the original three fudgeflakes is 1, what is the area of one of those three fudgeflakes? Justify your answer.



Complete, well-written solutions to at least two of the problems above, accompanied by a **Cover Sheet**, should be mailed to

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and **postmarked no later than January 5, 2003**. Each participant is expected to develop solutions without help from others. For the cover sheet and other details, see the USAMTS web site <http://www.nsa.gov/programs/mepp/usamts.html>.