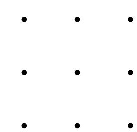
USA Mathematical Talent Search
PROBLEMS
Round 2 — Year 15 — Academic Year 2003–2004

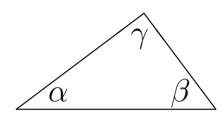
1/2/15. The faces of 27 unit cubes are painted red, white, and blue in such a manner that we can assemble them into three different configurations: a red $3 \times 3 \times 3$ cube, a white $3 \times 3 \times 3$ cube, and a blue $3 \times 3 \times 3$ cube. Determine, with proof, the number of unit cubes on whose faces all three colors appear.

2/2/15. For any positive integer n, let $s(n)$ denote the sum of the digits of n in base 10. Find, with proof, the largest n for which $n = 7s(n)$.

3/2/15. How many circles in the plane contain at least three of the nine points $(0,0)$, $(0,1)$, $(0,2)$, $(1,0)$, $(1,1)$, $(1,2)$, $(2,0)$, $(2,1)$, $(2,2)$? Rigorously verify that no circle was skipped or counted more than once in the result.



4/2/15. In how many ways can one choose three angle sizes, α, β, and γ, with $\alpha \le \beta \le \gamma$ from the set of integral degrees, $1°$, $2°$, $3°$, $\ldots$, $178°$, such that those angle sizes correspond to the angles of a nondegenerate triangle? How many of the resulting triangles are acute, right, and obtuse, respectively?



5/2/15. Clearly draw or describe a convex polyhedron that has exactly three pentagons among its faces and the fewest edges possible. Prove that the number of edges is a minimum.

Complete, well-written solutions to at least two of the problems above, accompanied by a Cover Sheet, should be mailed to:

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and postmarked by Sunday, 23 November 2003. Each participant is expected to develop solutions without help from others. For the cover sheet and other details, see the USAMTS web site: http://www.nsa.gov/programs/mepp/usamts.html.