USA Mathematical Talent Search

Round 1 Grading Rubric

Year 32 — Academic Year 2020–2021

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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/1/32:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each incorrect entry. (For grading purposes, joined cells are considered to be a single entry.)

Problem 2/1/32:

1 point: Student sets up a proof by contradiction with at least some constructive progress.

1 point: Student computes the sum of all the entries in two useful ways.

1 point: Student manipulates the equation regarding the sum of the entries into a form that makes it considerably easier to show the desired contradiction.

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2 points: Student explains why the equation has no integer solutions. Award **1 point** of partial credit for additional significant constructive progress towards this conclusion.

Note: Award a total of **1 point** for the correct answer with at least some justification. If the student writes the correct answer with no justification, award **0 points**.

Note: Solutions in which the student incorrectly computed the sum of all the entries typically received at most **2 points**, even if students' subsequent reasoning was valid.

Problem 3/1/32:

1 point: Student includes an accurate and meaningful figure. If a student is missing a figure, typically award at most **4 points**. In exceptional cases, graders may deduct more points for the absence of a figure if this makes it very hard to determine the student's progress.

1 point: Student makes at least one meaningful observation with justification (e.g., identifying, with proof, congruent triangles such as $\triangle ADP \cong \triangle ZDP$).

1 point: Student completes the proof that $PQRS$ is a rectangle. If a student assumes without proof that $PQRS$ is a rectangle, award a total score of at most **3 points**. If the student solution doesn't rely on $PQRS$ being a rectangle, the student can still receive **5 points** if their solution is complete and correct.

1 point: Student recognizes that $\triangle ZYS \sim \triangle DZP$ with justification or makes equivalent constructive progress.

1 point: Student completes the solution by determining that $\frac{AB}{BC} = 2 + \sqrt{3}$. If the student writes the correct answer with no explanation, award a total score of **1 point**.

Note: If a student incorrectly assumes WLOG that $ABCD$ is a rectangle (or another specific type of parallelogram), award at most **2 points**.

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Problem 4/1/32:

Note: Some students showed that the game must last exactly n moves, similar to the official solution. Other students used a "winning strategy" solution that shows that Rosencrans has a winning strategy for odd n and Gildenstern has a winning strategy for even n. Separate rubrics for each type of solution are as follows:

Official solution:

1 point: Student claims that the game lasts exactly n moves.

1 point: Student recognizes that it is useful to think in terms of parallel classes of chords and makes at least one useful observation accordingly (e.g., stating Lemma 1 or Lemma 2 without proof).

1 point: Student proves Lemma 1.

2 points: Student proves Lemma 2. Award **1 point** of partial credit for significant constructive progress towards this result. To receive this point of partial credit, it is sufficient to show the construction of $\overline{w,x}$ and explain why this chord has a shared point with both $\overline{s,t}$ and $\overline{u,v}$.

Winning strategy solutions:

1 point: Student claims that the game lasts exactly n moves, or (roughly) equivalently, Rosencrans has a winning strategy for odd values of n and Gildenstern has a winning strategy for even values of n.

2 points: Student describes a winning strategy for Rosencrans for odd n and explains why the winning strategy works regardless of Gildenstern's strategy. Award **1 point** of partial credit for significant constructive progress towards this result. To get the point of partial credit, it is sufficient to describe the winning strategy clearly.

2 points: Student describes a winning strategy for Gildenstern for even n and explains why the winning strategy works regardless of Rosencrans's strategy. Award **1 point** of partial credit for significant constructive progress towards this result. To get the point of partial credit, it is sufficient to describe the winning strategy clearly. The case in which n is even is more complicated than the case in which n is odd, and a common error was for participants to overlook a possible game scenario.

Problem 5/1/32:

1 point: Student uses the fact that a and b are rational to obtain a useful intermediate expression, such as $a = \left(\frac{m}{n}\right)^{\frac{m}{n-m}}$ in the official solution.

1 point: Student recognizes that $m - n = 1$ gives us a valid solution.

3 points: Student recognizes with proof that there are no additional solutions if we have $m - n > 1$. Award partial credit for significant constructive progress towards this result. Award **1 point** of partial credit if the student explains why both m and n must be perfect k^{th} powers. Award **2 points** of partial credit if the student applies the Binomial Theorem or uses some other promising method, but doesn't quite achieve the desired contradiction.

Note: Award a score of **1 point** for the correct answer, with or without justification.

Note: We did not award any points for finding the solution $(a, b) = \left(\frac{1}{4}, \frac{1}{2}\right)$ unless the student also made meaningful progress towards finding the general formula.

Note: Some students cited a paper: Sved, Marta, "On the rational solutions of $x^y - y^x$", Mathematics Magazine 63(1990), No, 1, 30-33. Even though this paper essentially solves the problem, we typically awarded **5 points** for these solutions, since the USAMTS rules encourage students to conduct mathematical research.