USA Mathematical Talent Search

Round 1 Problems

Year 18 — Academic Year 2006–2007

www.usamts.org

Please follow the rules below to ensure that your paper is graded properly.

1. You must show your work and prove your answers on all problems. If you just send a numerical answer for a problem with no proof, you will get no more than 1 point.

2. If you have not already sent an Entry Form, download an Entry Form from the Forms page at

   `http://www.usamts.org/MyUSAMTS/U_MyForms.php`

   and submit the completed form with your solutions.

3. If you have already sent in an Entry Form and a Permission Form, you do not need to resend them.

4. Put your name and USAMTS ID# on **every page you submit**.

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging into the site, then clicking on My USAMTS on the sidebar, then click Profile. If you are registered for the USAMTS and haven't received any email from us about the USAMTS, your email address is probably wrong in your Profile.

7. Do not fax solutions written in pencil.

8. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.

9. By the end of October, Round 1 results will be posted at www.usamts.org. To see your results, log in to the USAMTS page, then go to My USAMTS. Check that your email address in your USAMTS Profile is correct; you will receive an email when the scores are available.

10. Submit your solutions by October 10, 2006 (postmark deadline), via one (and only one!) of the methods below.

    (a) Email: solutions@usamts.org. Please see usamts.org for a list of acceptable file types. Do not send .doc Microsoft Word files.

    (b) Fax: (619) 445-2379 (Please include a cover sheet indicating the number of pages you are faxing, your name, and your User ID.)

    (c) Snail mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.

11. Re–read Items 1–10.

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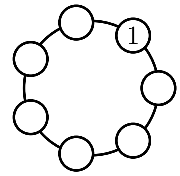
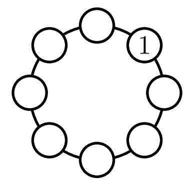
1/1/18. When we perform a 'digit slide' on a number, we move its units digit to the front of the number. For example, the result of a 'digit slide' on 6471 is 1647. What is the smallest positive integer with 4 as its units digit such that the result of a 'digit slide' on the number equals 4 times the number?

2/1/18.

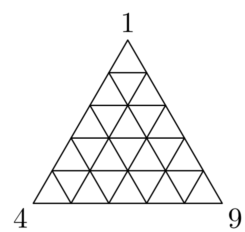


(a) In how many different ways can the six empty circles in the diagram at right be filled in with the numbers 2 through 7 such that each number is used once, and each number is either greater than both its neighbors, or less than both its neighbors?



(b) In how many different ways can the seven empty circles in the diagram at right be filled in with the numbers 2 through 8 such that each number is used once, and each number is either greater than both its neighbors, or less than both its neighbors?

3/1/18.



(a) An equilateral triangle is divided into 25 congruent smaller equilateral triangles, as shown. Each of the 21 vertices is labeled with a number such that for any three consecutive vertices on a line segment, their labels form an arithmetic sequence. The vertices of the original equilateral triangle are labeled 1, 4, and 9. Find the sum of the 21 labels.

(b) Generalize part (a) by finding the sum of the labels when there are n^2 smaller congruent equilateral triangles, and the labels of the original equilateral triangle are a, b, and c.

4/1/18. Every point in the plane is colored either red, green, or blue. Prove that there exists a rectangle in the plane such that all four of its vertices are the same color.

5/1/18. $ABCD$ is a tetrahedron such that $AB = 6$, $BC = 8$, $AC = AD = 10$, and $BD = CD = 12$. Plane $\mathcal{P}$ is parallel to face ABC and divides the tetrahedron into two pieces of equal volume. Plane $\mathcal{Q}$ is parallel to face DBC and also divides $ABCD$ into two pieces of equal volume. Line ℓ is the intersection of planes $\mathcal{P}$ and $\mathcal{Q}$. Find the length of the portion of ℓ that is inside $ABCD$.

Round 1 Solutions must be submitted by **October 10, 2006**.
Please visit **http://www.usamts.org** for details about solution submission.