USA Mathematical Talent Search

Round 1 Problems

Year 20 — Academic Year 2008–2009

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer for a problem with no proof, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by October 14, 2008, via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)

   (b) Mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090. (Solutions must be postmarked on or before the deadline.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

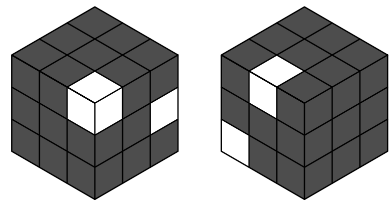
6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto www.usamts.org and visiting the "My USAMTS" pages. (If you are registered for the USAMTS and haven't received any email from us about the USAMTS, then your email address is probably wrong in your Profile.)

7. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on www.usamts.org.

Each problem is worth 5 points.

1/1/20. 27 unit cubes—25 of which are colored black and 2 of which are colored white—are assembled to form a $3 \times 3 \times 3$ cube. How many distinguishable cubes can be formed? (Two cubes are indistinguishable if one of them can be rotated to appear identical to the other. An example of two indistinguishable cubes is shown at right.)



2/1/20. Find all positive integers n for which it is possible to find three positive factors x, y, and z of $n-1$, with $x > y > z$, such that $x + y + z = n$.

3/1/20. Let a, b, c be real numbers. Suppose that for all real numbers x such that $|x| \leq 1$, we have $|ax^2 + bx + c| \leq 100$. Determine the maximum possible value of $|a| + |b| + |c|$.

4/1/20. A point P inside a regular tetrahedron $ABCD$ is such that $PA = PB = \sqrt{11}$ and $PC = PD = \sqrt{17}$. What is the side length of $ABCD$?

5/1/20. Call a positive real number *groovy* if it can be written in the form $\sqrt{n} + \sqrt{n+1}$ for some positive integer n. Show that if x is groovy, then for any positive integer r, the number x^r is groovy as well.